Quantum Mechanics from the Energy Circulation Theory

Wavefunction showing an energy location in real 3D space

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Summary

- Elementary circulation (particle) at rest:

  Intrinsic energy m_0 is circulating at $v_c = \mu_0\omega_0 = c$. $E_0 = m_0{\mu_0}^2{\omega_0}^2$

- Its linear motion:

  Receiving a force (free motion), the linear velocity can continuously alter.

  $E = m_0c^2 = m_0\left({C_r}^2 + v^2\right)$ m_0 is helically moving at c.

- If a linear motion becomes circle (orbiting):

  Frequency ω of internal circulation and Ω of orbiting become quantized.

$$\omega = n\Omega\,, \qquad (n\text{: integer})$$

- Atomic electron: Centrifugal and electric forces balance.

< Contradiction of the Schrödinger equation >

Energy operator, Momentum operator:

$$E_q \psi(x,t) = i\hbar \frac{\partial}{\partial t} \psi(x,t) \,, \qquad p_q \psi(x,t) = -\, i\hbar \frac{\partial}{\partial x} \psi(x,t)$$

Substitute the operators to $H = \frac{1}{2} m_r v^2 + V = \frac{p^2}{2m_r} + V$.

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = -\frac{\hbar^2}{2m_r} \frac{\partial^2}{\partial x^2} \psi(x,t) + V(r)\psi(x,t)$$

m_r: rest mass

Contradictions:

1) In obtaining energy and momentum operators, $E = pv = m_r v^2$ is used.
2) When substitute them to Hamiltonian equation, $E = pv/2 = p^2/2m_r$ is used.
3) Speed of atomic electron $v \approx c$: Approximation $E_k \approx m_r v^2/2$ is not valid.

$$E_k = -\frac{m_r c^2}{2} \log\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{2} m_r v^2 \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{1}{3}\frac{v^4}{c^4} + \frac{1}{4}\frac{v^6}{c^6} + \cdots\right)$$

< Expression of energy distribution >

Use the notation: $E\psi \Longrightarrow$ Energy E is located at ψ.

Circular motion:

$$\mu\varphi = \mu\exp(i\omega t) = \mu(\cos\omega t + i\sin\omega t) = [x_1 \quad x_2] = \mu[\cos\omega t \quad \sin\omega t]$$

Both x_1 and x_2 can be space dimensions.

Helical motion with a linear velocity v:

$$\psi = jvt + \mu\varphi = jvt + \mu\exp(i\omega t)$$

Use plane waves, $k = 2\pi/\lambda = \omega/v$:

$$\psi_1 = \mu\cos\big(k(x - vt)\big) = \mu\cos(kx - \omega t)$$

$$\psi_2 = i\mu\sin(kx - \omega t)$$

$$\psi = \psi_1 + \psi_2 = \mu\exp\big(i(kx - \omega t)\big)$$

The two equations of ψ are equivalent. Even if we use imaginary units i and j, it expresses a helical motion in the real 3D space.

< Quantization of a circulating particle >

Single circulation S at rest: $E_0\mu_0\varphi(\omega_0)$, $E_0 = m_0{\mu_0}^2{\omega_0}^2$

Quantized (n: integer): $E_n\mu_0\varphi(n\omega_0)$, $E_n = m_0{\mu_0}^2(n\omega_0)^2 = n^2 m_0{\mu_0}^2{\omega_0}^2$

m_0: minimum intrinsic energy moving at $\mu_0\omega_0 = c$, invariant.

μ_0: radius of a spacia, invariant.

Adding a force, make it move at v. m_0 moves helically at c.

$$\psi = jvt + \mu_0 \exp(i\omega t) \ , \quad E = m_0 c^2$$

$$c^2 = v^2 + {\mu_0}^2\omega^2 \ , \quad \omega = \omega_0\sqrt{1 - v^2/c^2}$$

When jvt gets circular, the internal circulation and orbiting are quantized.

$$\omega = n\Omega \ (n\text{: integer}) \ , \qquad \Omega = r/v$$

$$\omega = n\, r/v \ , \qquad v^2 = \frac{{\omega_0}^2 r^2}{n^2 + {\omega_0}^2 r^2/c^2}$$

Take the relative radius $R \equiv r/\mu_0$

$$v = \pm\frac{R}{\sqrt{n^2 + R^2}}c$$

Linear motion of a hidden-space dimensional single circulation iS:

In hidden dimension, quantized and localized in one layer of $2\mu_0$ width.

Location in i remains as $\mu_0\varphi_0 = \mu_0\exp(i\omega_0 t)$.

Elementary electric charge $e = m_0\mu_0\omega_0/2$ is invariant by v.

In space dimensions, we can express by the same equation as for S.

$$\psi = jvt + \mu_0\exp(i\omega t) \quad , \quad E = m_0c^2$$

We regard $\mu_0\exp(i\omega t)$ as a 1D space vibration instead of circulation.

Characteristics of the wavefunction ψ:

- ψ shows a location of energy in the real 3D space.
- ψ is decided only by the linear velocity v. Independent of value E.
- ψ is common for any types of energy of any particles.

$$E\psi \,, \qquad E_r\psi \,, \qquad E_k\psi \,, \qquad m\psi \,, \qquad p\psi$$

< Express ψ by energy quantum E_q and its momentum p_q >

Plane-wave expression: $\psi = \mu_0 \exp\big(i(kx - \omega t)\big)$

Expression of energy by rest energy (mass) and kinetic energy:

$E = m_r c^2 + \Delta E = m_r c^2 + E_k$ (treat a rest mass as invariant)

Expression by circular component and linear component energies:

$$E = m_r c^2 + \Delta E = mc^2 = m\left({C_r}^2 + v^2\right)$$

$m \equiv E/c^2$ is the "**mass**" different from the "**rest mass**" m_r.

Linear component energy:

$E_l = mv^2 = pv$ (valid for any quantity of m and v)

Energy quantum for a single cycle:

Energy of a photon (single cycle of light) and Planck constant:

$$E_p = \frac{E_\gamma}{\nu} = \frac{2\pi}{\omega} \frac{E_{(iS)}}{2{\omega_0}^2} \omega^2 = \frac{\pi E_0}{{\omega_0}^2} \omega = \frac{2\pi^2 E_0}{{\omega_0}^2} \nu \equiv h\nu$$

$$h = \frac{2\pi^2 E_0}{{\omega_0}^2} = \frac{2\pi^2 m_0 {\mu_0}^2 {\omega_0}^2}{{\omega_0}^2} = 2\pi^2 m_0 {\mu_0}^2$$

If we express E by co-moving frame, $E = M{C_r}^2$.

$$E = mc^2 = M{C_r}^2 = M{\mu_0}^2\omega^2 = 4\pi^2 M{\mu_0}^2\nu^2$$

$$E = \frac{2M}{m_0}h\nu^2$$

Energy per cycle can be expressed by energy quantum $E_q \equiv h\nu$.

$$\frac{E}{\nu} = \frac{2M}{m_0}h\nu = nE_q$$

Momentum of E_q for a linear component energy: $p_q \equiv m_q v$

$$E_q = m_q v^2 = p_q v\,, \qquad p_q = E_q/v = \hbar\omega/v$$

$$p_q = \hbar k \;\;, \quad (\omega = kv)$$

The relation $p = \hbar k$ is valid only for an **energy quantum** of

a **linear component energy**.

We get the wavefunction by p_q and E_q from $k = p_q/\hbar$ and $\omega = E_q/\hbar$.

$$\psi(x,t) = \mu_0 \exp\!\big(i\,(p_q x - E_q t)/\hbar\big)$$

< Derive a wave equation to have a solution ψ >

$$\psi(x,t) = \mu_0 \exp\left(i\left(p_q x - E_q t\right)/\hbar\right)$$

Take the partial differentials by t and x:

$$\frac{\partial}{\partial t}\psi(x,t) = -i\frac{E_q}{\hbar}\psi(x,t)$$

$$\frac{\partial}{\partial x}\psi(x,t) = i\frac{p_q}{\hbar}\psi(x,t)$$

Multiply both sides by i.

$$E_q\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t) \quad \text{(Energy operator)}$$

$$p_q\psi(x,t) = -\,i\hbar\frac{\partial}{\partial x}\psi(x,t) \quad \text{(Momentum operator)}$$

Redefine the mechanical energy: linear component energy + potential energy

$$E_m = mv^2 + V(r) = \frac{p^2}{m} + V(r)$$

By substituting the operators to the equation, we get the wave equation.

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{m_q}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(r)\psi(x,t)$$

$$m_q = E_q/c^2 = \hbar\omega/c^2$$

The equation is meaningful for a circular motion. x is in arc direction. r is the radius. The wavefunction ψ is a solution of it.

$$\psi = \mu_0 \exp\big(i(kx - \omega t)\big) = \mu_0 \exp\left(i\frac{\omega}{v}(x - vt)\right)$$

Difference from the Schrödinger equation:

- The mass is not a rest mass of a particle, but a mass of an energy quantum, decided by v.
- The frequency ω is independent of the quantity of mass, only decided by v.
- Solution is applicable to any energy types because the location is common.

$$E_t\ , E_c\ , E_l\ , E_r\ , E_k\ , m\ , p$$

< Circular motion of an atomic electron >

Electron: $e^{-}(H, iH_{-})$ adduct of hemi-circulations (H, iH_{-})

H: $S \longrightarrow \nu(H) + \overline{\nu}(\overline{H})$ $E_{(H)} = E_0/2 = m_0c^2/2$

iS: $E_{(iS)}\mu_0 \exp(i\omega_0 t) = E_0[X \quad H] = E_0\mu_0[\cos\omega_0 t \quad \sin\omega_0 t]$

Prolongation of iS: $iS + \Delta E \rightleftarrows iH_{+} \cdots iH_{-}$

iS prolongs to n circulations in n spacias.

Momentums in H, equal to $+e$ and $-e$, are preserved.

Forces set off as 0 at medial junctions. Only two ends affect attraction.

$$F_x = K_e \frac{(e/n)(-e/n)}{(2\mu_0)^2} = -K_e \frac{e^2}{(2n\mu_0)^2} = -K_e \frac{e^2}{d^2}$$

For orbiting direction, the energy of electron is only on the minus-end spacia. In fact, it is contributed only by a neutrino.

$$E_{(e)} = \frac{m_0c^2}{2} + \frac{E_{(n-iS)}}{2n} \approx \frac{m_0c^2}{2}, \qquad (n \approx 10^4)$$

For radial direction, the energy of electron is the sum of a half of prolonged iS and a neutrino.

$$E_{(n-iS)} = m_0 c^2 + \Delta E$$

$$E_e(r) = \frac{m_0 c^2}{2} + \frac{E_{(n-iS)}}{2} = m_0 c^2 + \frac{\Delta E}{2}$$

Added energy to electron is the difference in electric potential energy.

$$\frac{\Delta E}{2} = -\frac{1}{2}\int_{2\mu_0}^{r} F_e dr = V(r) - V(2\mu_0)$$

$$E_e(r) = m_0 c^2 + \ V(r) - V(2\mu_0)$$

$$E_e(r) - E_e(r_0) = V(r) - V(r_0)$$

Circulation of an electron in xy can rotate in yz.

Condition of quantization:

For a single circulation: $\omega = n\Omega$

Same as Bohr's formula $mvr = nh/2\pi$ ($2\pi r = n\lambda$, $\lambda = h/p$ de Broglie wave)

For multiple m circulations (including rotation):

$$\omega_{nm} = nm\Omega_{nm} \quad (n = 1, 2 \ \cdots) \ (m = 1, 2 \ \cdots \leq n)$$

Velocity of electron v:

Even if ω varies by multiple times, v is close to light speed c.

$$\omega = \omega_0\sqrt{1 - v^2/c^2}\ , \qquad v = \pm\frac{R}{\sqrt{n^2 + R^2}}c\ , (R \approx 10^4)$$

We can regard v as constant for the centrifugal force, while it varies for ω.

Orbiting speed v_{orb}: $$v_{orb} = r_{nm}\Omega_{nm} = v/m$$

Centrifugal force balances with electric force.

$$\frac{m_e {v_{orb}}^2}{r_{nm}} = \frac{K_e e^2}{{r_{nm}}^2}\ , \qquad r_{nm} = \frac{K_e e^2}{m_e}\cdot\frac{1}{{v_{orb}}^2}$$

Radius of orbiting:

$$r_{nm} = \frac{K_e e^2}{m_e}\cdot\frac{m^2}{v^2} \approx \frac{K_e e^2}{m_e c^2}m^2 \equiv K_r m^2$$

Proportional to m^2 but independent of n.

< Wavefunction of an atomic electron >

3D wave equation:

$$i\hbar\frac{\partial}{\partial t}\psi(x,y,z,t) = -\frac{\hbar^2}{m_q}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(x,y,z,t) + V(x,y,z)\psi(x,y,z,t)$$

Time-independent equation:

$$\psi(x,y,z,t) = \mu_0 \exp\left(i\left(k\sqrt{x^2+y^2+z^2}-\omega t\right)\right) = \psi(x,y,z)\exp(-i\omega t)$$

Substitute it to the above wave equation.

$$E_q\psi(x,y,z) = -\frac{\hbar^2}{m_q}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(x,y,z) + V(x,y,z)\psi(x,y,z)$$

$\psi(x,y,z)$ does not show a stationary wave, but its amplitude.

In electron orbiting, the frequency $\mathbf{\Omega}$ is a **vector**. Express $\psi(x,y,z,t)$ by location-dependent and time-dependent parts:

$$\psi(x,y,z,t) = R(x,y,z)\exp(-i\mathbf{\Omega}t) = \left(r_x\mathbf{e_x}+r_y\mathbf{e_y}+r_z\mathbf{e_z}\right)\exp(-i\mathbf{\Omega}t)$$

$$= \left[r_x\exp(-i\Omega_x t) \quad r_y\exp\left(-i\Omega_y t\right) \quad r_z\exp(-i\Omega_z t)\right]$$

< Orbitals of an atomic electron >

1) $n = 1$, $m = 1$: 1S orbital

$$\psi_{1S} = r_{1S}\varphi_{xy}(\Omega_{11}) = r_{1S}\exp(i\Omega_{11}t) = r_{1S}[\cos\Omega_{11}t \quad \sin\Omega_{11}t]$$

$$= \left(r_{1S}\mathbf{e_x} + r_{1S}\mathbf{e_y} + 0\mathbf{e_z}\right)\left(\exp(i\Omega_{11}t)\mathbf{e_x} - i\exp(i\Omega_{11}t)\mathbf{e_y} + \mathbf{e_z}\right)$$

$$\equiv R(x,y,z)\exp(i\boldsymbol{\Omega}_{\mathbf{11}}t) \qquad \boldsymbol{\Omega}_{\mathbf{11}} = [\Omega_x \quad \Omega_y \quad \Omega_z] = [\Omega_{11} \quad \Omega_{11} \quad 0]$$

$$\omega_{11} = \Omega_{11}$$

Wave function ψ_{1S}:

- Obtained by radius r_{1S} and circular function $\varphi_{xy}(\Omega_{11})$, a 2D circulation.
- Product of $R(x,y,z)$ and $\exp(i\boldsymbol{\Omega}_{\mathbf{11}}t)$
- Solution of the wave equation for $\omega = \omega_{11} = \Omega_{11}$

By standard quantum mechanics:

- Treat v and dr/dt as variant.
- Solution of the time-independent equation for 1S is $\psi_{1S}(r) = A_{1S}\exp(-r/a_0)$. Give it the radial symmetry. Distribution in sphere of r is $4\pi r^2[\psi_{1S}(r)]^2$, which shows the maximum at $r = a_0$ (Bohr's radius)
- We cannot give the radial symmetry to a solution $\psi(x,y,z)$ unconditionally.

2) $n = 2$, $m = 1$: 2S orbital

$$\psi_{2S} = r_{2S}\varphi_{xy}(\Omega_{21}) = r_{2S}[\cos\Omega_{21}t \quad \sin\Omega_{21}t]$$

$$r_{2S} = r_{1S}\,, \qquad \omega_{21} = nm\Omega_{21} = 2\Omega_{21}$$

Radius of S orbitals:

- $r_{2S} = r_{1S}$ because $m = 1$ for both.
- Bigger the atomic number is, greater the effective positive charge for 1S electron is. r_{1S} is larger if the atomic number is bigger.

3) $n = 2$, $m = 2$: 2P orbital

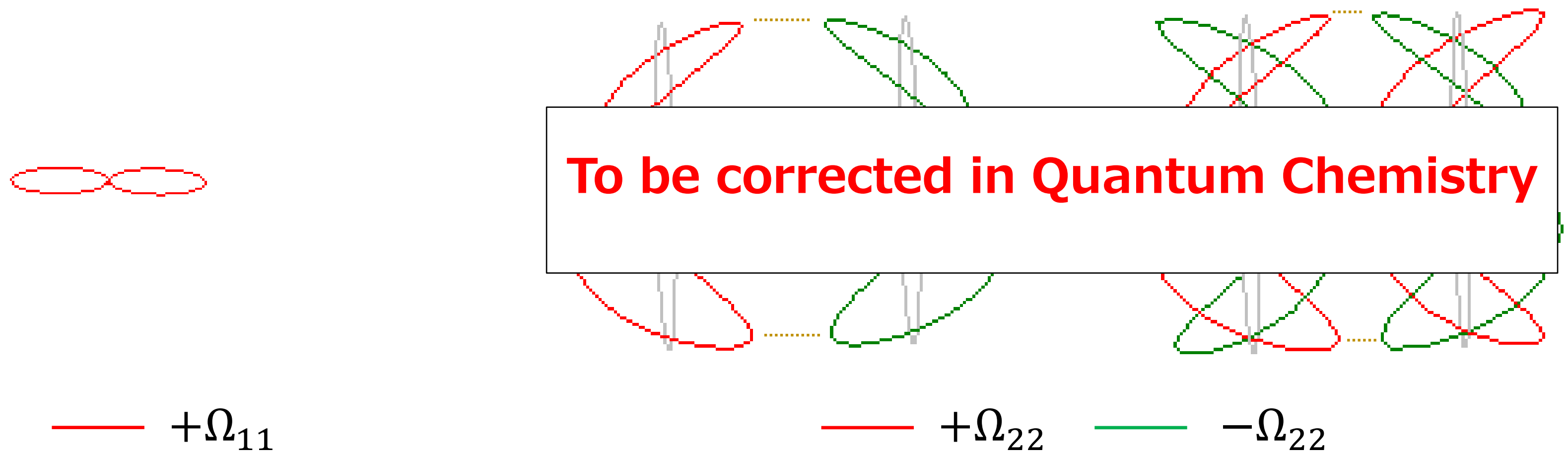
$\psi_{2P} = r_{2P}$

$= r_{2P}$

To be corrected in Quantum Chemistry

$$r_{2P} = m^2 r_{2S} = 4r_{2S}\,, \qquad \omega_{22} = nm\Omega_{22} = 4\Omega_{22}$$

- Circulation in xy by $\varphi_{xy}(\Omega_{22})$ rotates in yz by $\varphi_{yz}(\Omega_{22})$.
- Electron is circulating in xy by Ω_{22} with vibrating in y by $2\Omega_{22}$.
- Radial velocity is zero $dr/dt = 0$ even in P orbitals.



(a) 1S-1S sigma bond (b) Pi bond by 2P-2P in C=C (c) Two pi bonds in C≡C

Atomic orbitals and their bindings. (a) Sigma bond by two 1S orbitals in H_2. (b) Pi bond by two 2P orbitals in C=C double bond. (c) Two pi bonds in C≡C triple bond. (a) is by flat interaction of same directional circulations. (b) and (c) are by orthogonal interaction of opposite directional circulations.

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Energy circulation theory (ECT):

ECT home: https://mitiempo-ect.main.jp/ECT.html