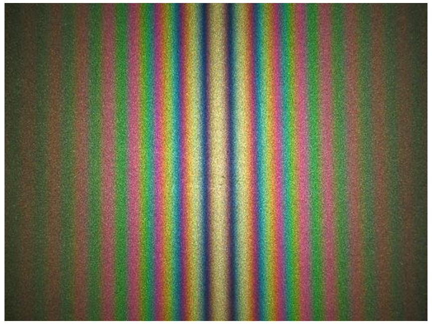
Michelson-Morley experiment

(anisotropy of light speed)

Michelson-Morley (M-M) experiment

- did not detect ***shift*** of **interference fringe** by rotation or different time in day or year,
- is said to be a successful experiment that has proved the **absence** of the **light medium** (luminiferous aether).



Fringe pattern by white light (from Wikipedia)

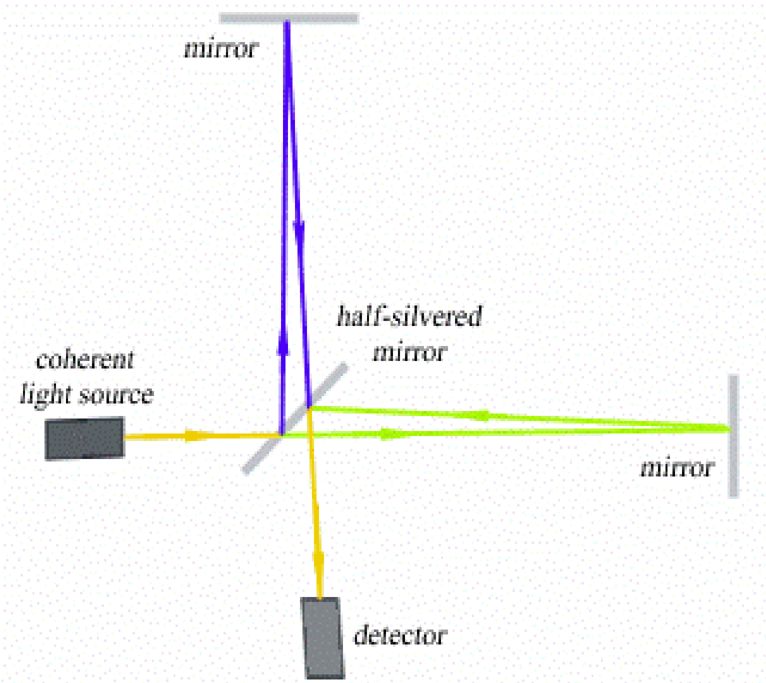
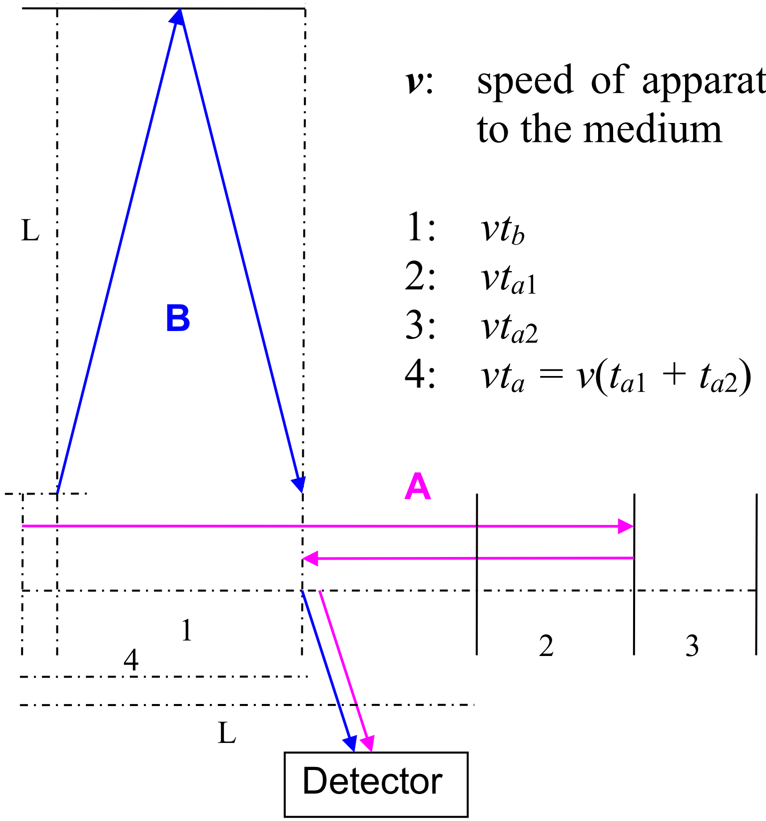


Figure S-10-2. Michelson-Morley experiment (from Wikipedia)

Is the **experimental design** indeed **capable** to detect the interference **fringe shift**?

M-M experiment by the frame stationary to the medium



v: speed of apparatus to the medium

1: vt_b
2: vt_{a1}
3: vt_{a2}
4: $vt_a = v(t_{a1} + t_{a2})$

- Phase propagation speed is constant independent of the speed of the emitter to the medium. The light speed is constant by the medium's frame.
- The route of **Beam A** parallel to the apparatus movement is longer than the route of **Beam B** for the round trip between the half-mirror and the end mirror.

$$D_a = L + \sqrt{L^2 + v^2 t_a{}^2}^{\,*} \quad , \quad D_b = \sqrt{4L^2 + v^2 t_b{}^2}$$

Because light speed is equal for both beams,

$$c = D_b / t_b = D_a / t_a \ . \qquad t_b{}^2 = \frac{2Lt_a{}^2}{L + \sqrt{L^2 + v^2 t_a{}^2}}$$

$$t_b < t_a \ \text{ if } \ v \neq 0 . \qquad \Delta t \equiv t_a - t_b$$

* From $D_a = L + vt_{a1} + L - vt_{a2} = 2L + 2vt_{a1} - vt_a$ and $t_a / t_{a1} = D_a / (L + vt_{a1})$, t_{a1} is eliminated.

Displacement of Beam A is released at the splitter Δt **earlier** than that of Beam B so that the two reach the same point at the same instance for interference detection.

M-M experiment by Medium's frame (continued)

Propagation of the combined beam (A+B) from the **combined point** to the **detector**:

$$\varphi_\perp + \varphi_\parallel = A\sin(kx - \omega t) + A\sin\big(kx - \omega(t + \Delta t)\big)$$

$$= A\sin\left(kx - \omega t - \frac{\omega\Delta t}{2} + \frac{\omega\Delta t}{2}\right) + A\sin\left(kx - \omega t - \frac{\omega\Delta t}{2} - \frac{\omega\Delta t}{2}\right)$$

$$= 2A\cos\frac{\omega\Delta t}{2}\exp\left(i\left(kx - \omega t - \frac{\omega\Delta t}{2}\right)\right)$$

A : Maximum amplitude of Beam A and Beam B

x : Distance from the combined point in the direction to the interferometer

ω : Angular frequency

k : Angular wave number $\quad k = \omega/c$

By the **variation** of Δt there is **no change** of **frequency**, **wavelength** or **wave number**, while the **maximum amplitude** and the **phase** vary.

M-M experiment by the frame fixed to the apparatus

- Light travel distance: Same for both Beam A and Beam B ($2L$)
- Time: Same as time by the medium's frame
- Average light speed for round trip between half-mirror and end-mirror:

  Different between Beam A and Beam B shown by $c_A = 2L/(t+\Delta t)$, $c_B = 2L/t$

  Light speed by moving frame: $c_v = c - v\cos\theta$ (θ : angle between the light propagation and $\boldsymbol{v}$)
- **Direction of light propagation at the detector**: **Identical** for Beam A and Beam B
- Therefore, **light speed at the detector**: **Common** for Beam A and Beam B

Combined wave propagation to the detector by apparatus' frame:

$$\varphi'_{\perp} + \varphi'_{\parallel} = 2A\cos\frac{\omega\Delta t}{2}\exp\left(i\left(k'x' - \omega t - \frac{\omega\Delta t}{2}\right)\right)$$

$$k' = \omega/c'$$

$\varphi'_{\perp}, \varphi'_{\parallel}, k', x'$: measured by the apparatus' frame

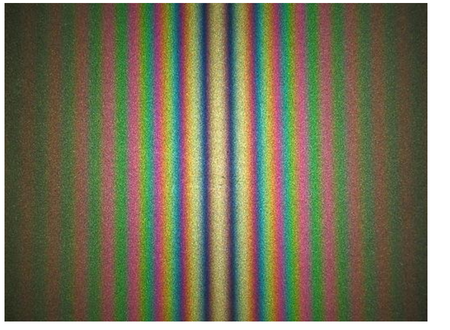
c': Light speed by the apparatus' frame in the direction of x'

Conclusion by apparatus' frame is same as that by medium's frame:

No change of frequency, wavelength or wave number by variation of Δt

M-M experiment: Interference fringe

- The combined wave goes into interferometer, and passes through two slits with a very small distance.
- On the detection screen, interference fringe is shown.
- The location and pattern of the fringe depend on the slit pitch and the distance from the slits to screen.
- The fringe pattern varies by light frequency or wavelength, but is not altered by the variation of the phase of light.



Fringe pattern by white light (from Wikipedia)

Conclusion of Michelson-Morley type experiments:

> By the **variation** of Δt, **no change** of **position** of interference fringe should be detected while the **brightness** of the fringe would be altered.

Reason why M-M like experiments are incapable to detect the interference fringe shift:

The two beams are finally combined in a ***common direction*** for measurement of interaction.