

# ***Hubble Diagram from the 4D Spherical Universe***

## ***(Nature of the time and 4DS model - Part 3)***

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2. 4-D Spherical Model of the universe
3. Michelson-Morley experiment
4. Acceleration factor
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## Definition of terms related to Time

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<b><math>x</math></b> (Radius of universe):	Radius of the 4D sphere of universe, equal to the Observed Time ( $T$ )
<b>CU</b> (Cosmic Unit):	Unit of $x$ and $T$ , being 1 at its maximum when the space expansion stops. Time-related variables are expressed in the CU.
<b><math>T_E</math></b> (Time of Emission):	Time when the light was emitted.
<b><math>T_P</math></b> (Present Time):	Present Time of universe, when the light reaches us.
<b><math>T_{ER}</math></b> (Relative Time of Emission):	Relative ratio of $T_E$ to $T_P$ . $T_{ER} \equiv T_E / T_P$
<b><math>T_B</math></b> (Back in Time):	Back in Time from present when the light was emitted. $T_B \equiv T_P - T_E$
<b><math>T_{BR}</math></b> (Relative Back in Time):	Relative ratio of $T_B$ to $T_P$ . $T_{BR} \equiv T_B / T_P$
<b><math>T_C</math></b> (Time Clear):	Time when the space became transparent to light.
<b><math>T_{CR}</math></b> (Relative Time Clear):	Relative ratio of $T_C$ to $T_P$ . $T_{CR} \equiv T_C / T_P$

**$t$ , time** (Original time),  **$T$ , Time** (Observed Time): See for definition in the Part 1.



# Redshift from values at the emission

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## Wavelength-based redshift:

By **space expansion**: Wavelength is stretched by  $n$ .

$$z_{\lambda}^e + 1 = \frac{\lambda(T_P)}{\lambda_0(T_E)} = n = \frac{T_P}{T_E} = \frac{1}{T_{ER}}$$

## Frequency-based redshift:

$$z_{\nu}^e + 1 = \frac{\nu_0(T_E)}{\nu(T_P)} = \frac{C(T_E)}{\lambda_0(T_E)} \frac{\lambda(T_P)}{C(T_P)} = n \frac{C(T_E)}{C(T_P)} = \frac{1}{T_{ER}} \frac{C(T_E)}{C(T_P)}$$

$n$ : Expansion ratio of universe

$T$ : Observed Time = Radius of universe

# Redshift of actual measurement

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From the **energy circulation theory**:

Atomic energy varies by the space expansion.

Energy of the single circulation  $iS$  in hidden-space dimensions:

$$E_{(iS)} = m_0 c^2 = m_0 v_c^2 = m_0 \mu_0^2 \omega_0^2$$

Light energy / Photon energy:

$$E_\gamma = h\nu^2 = \hbar\omega^2, \quad E_p = E_\gamma/\nu = h\nu = \hbar\omega$$

Space expansion by  $n \Rightarrow E_{(iS)}, c^2$  and  $\omega_0^2$  decrease by  $1/n^3$ . ( $m_0, \mu_0$ : invariant)

Present atom of the same element: Light energy decreases by  $1/n^3$  at emission.

$$E_\gamma(T_E) = h\nu_{0(T_E)}^2 \propto m_0 C_{(T_E)}^2, \quad E_\gamma(T_P) = h\nu_{0(T_P)}^2 \propto m_0 C_{(T_P)}^2$$

$$\frac{\nu_0(T_P)}{\nu_0(T_E)} = \frac{C(T_P)}{C(T_E)}$$

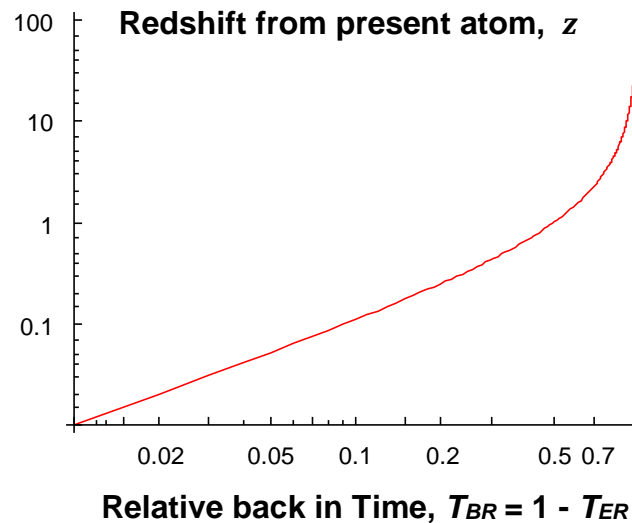
## Redshift from the present atom

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$$z_\lambda + 1 = \frac{\lambda(T_P)}{\lambda_0(T_P)} = \frac{\lambda(T_P)}{\lambda_0(T_E)} \times \frac{\lambda_0(T_E)}{\lambda_0(T_P)} = n \times \frac{C(T_E) \nu_0(T_P)}{\nu_0(T_E) C(T_P)} = n = \frac{1}{T_{ER}}$$

$$z_\nu + 1 = \frac{\nu_0(T_P)}{\nu(T_P)} = \frac{\nu_0(T_E)}{\nu(T_P)} \times \frac{\nu_0(T_P)}{\nu_0(T_E)} = n \frac{C(T_E)}{C(T_P)} \times \frac{C(T_P)}{C(T_E)} = n = \frac{1}{T_{ER}}$$

$$z_\lambda = z_\nu = z = \frac{1}{T_{ER}} - 1$$



## 10. Factors affecting the brightness

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### (i) Factor by **wavelength prolongation**

Along with space expansion:

- Wavelength: stretched by  $n \equiv T_P/T_E = 1/T_{ER}$
- Energy of a single photon  $h\nu$ : decreases to  $1/n$
- Number of photons: increase to  $n$ -fold (preservation of the total energy)
- $h\nu$  times the number of photon: no change
- **No effect by wavelength prolongation on the luminosity** (total energy per unit time)

### (ii) Factors by **scattering**

Light speed: 
$$C(x) = K * f_D * f_{EM} = K * \frac{1}{x\sqrt{1-x}} * \left(1 - \frac{T_C^3}{x^3}\right)$$

- Electromagnetic interaction factor  $f_{EM}$  is effect of scattering.
- Plural Time Clear  $T_C$  values:
  - For Cosmic Microwave Background radiation: 380,000 years after Big Bang
  - For “Reionization” of interstellar hydrogen by stars: 150 - 1,000 million years after Big Bang
  - For light propagation in substances
- If the density is constant, the transmittance is  $I(x) = I_0 \left(1 - \int (T_C \rho)^3 dx\right) = I_0(1 - kx)$ .
- Light from stars emitted during the reionization period: Not reach us or be darken by scattering
- Stellar light emitted later ( $T_{ER} > 0.072$ ,  $z < 12.8$ ): **Factor by scattering is negligible.**

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(iii) Factor by **variation of the space energy density**

- Luminosity: energy per unit-time

Flux: energy per unit-time per unit-area

$$F = \frac{L}{4\pi r^2}$$

- Luminosity varies depending on the light speed at the time of detection because it is a per unit-time value.  $L$ : function of the time of detection  $T = x$

$$F(x) = \frac{L(C(x))}{4\pi r^2} = \frac{L(x)}{4\pi r^2}$$

- For our observation, the variable  $x$  is fixed to the present time  $T_P$ . **Luminosity and Flux are invariant by the time of emission  $T_E$ .**
- **No effect** by variation of the space energy density on brightness for our observation.



# 11. Hubble diagram – Magnitude of flux and distance

Light of luminosity  $L$  was emitted at  $T_E$  and reaches us now at  $T_P$ .

Flux we observe now:

$$F(T_E) = \frac{L}{4\pi \cdot LD(T_E)^2}$$

Define its magnitude:

$$m(T_E) \equiv -2.5 \cdot \lg F(T_E)$$

Define the **relative magnitude to the same luminosity with  $z = 0.05$** :

$$DM_{0.05}(T_{ER}) \equiv m(T_{ER}) - m(z = 0.05) = m(T_{ER}) - m(1/1.05)$$

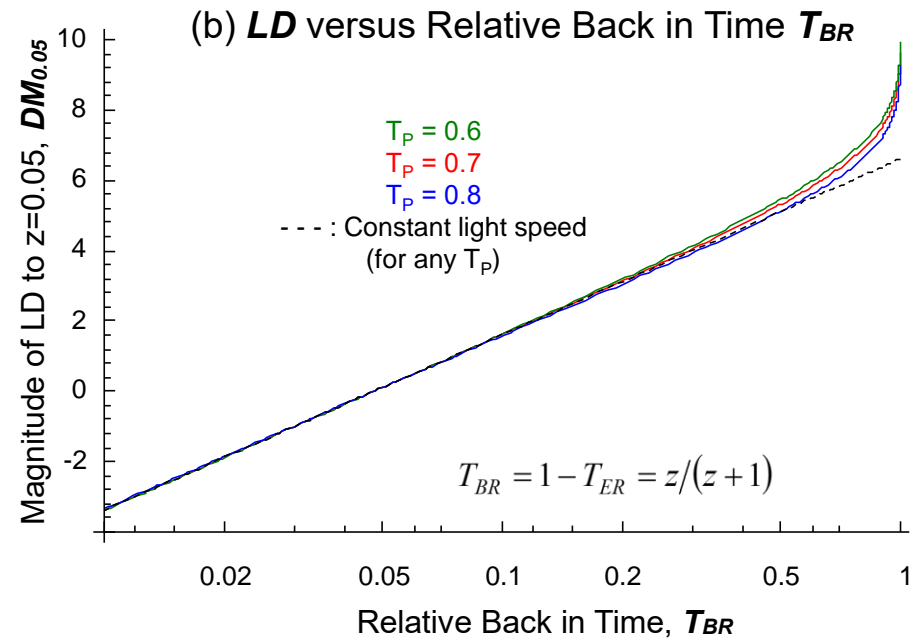
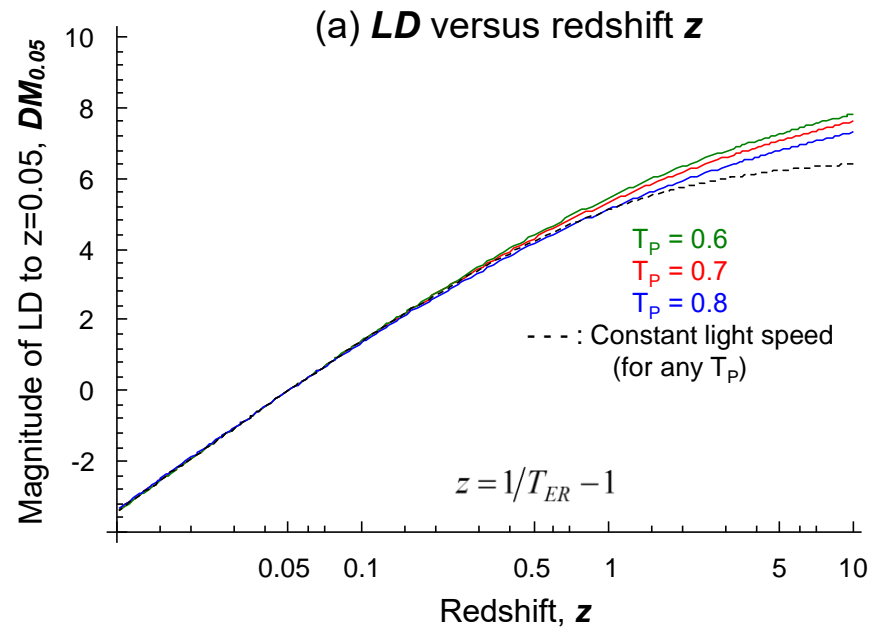
- This is a **distance modulus** : referred to as the “**magnitude of LD to z=0.05**”
- Light propagated distance  $LD$ :  $f_{EM}$  can be ignored if  $z < 12.8$

$$LD(T_E) \approx \int_{T_E}^{T_P} \frac{K}{x\sqrt{1-x}} dx = K \left( \log \left( \frac{1 - \sqrt{1-T_P}}{1 + \sqrt{1-T_P}} \right) - \log \left( \frac{1 - \sqrt{1-T_E}}{1 + \sqrt{1-T_E}} \right) \right)$$

$$DM_{0.05}(T_{ER}) = 5 \cdot \lg \left( \log \left( \frac{1 - \sqrt{1-T_P}}{1 + \sqrt{1-T_P}} \cdot \frac{1 + \sqrt{1-T_P T_{ER}}}{1 - \sqrt{1-T_P T_{ER}}} \right) \right) - 5 \cdot \lg \left( \log \left( \frac{1 - \sqrt{1-T_P}}{1 + \sqrt{1-T_P}} \cdot \frac{1 + \sqrt{1-T_P/1.05}}{1 - \sqrt{1-T_P/1.05}} \right) \right)$$

# Magnitude of light propagated distance $LD$ to $z = 0.05$

$$DM_{0.05}(T_{ER}) = 5 \cdot \lg \left( \log \left( \frac{1 - \sqrt{1 - T_P}}{1 + \sqrt{1 - T_P}} \cdot \frac{1 + \sqrt{1 - T_P T_{ER}}}{1 - \sqrt{1 - T_P T_{ER}}} \right) \right) - 5 \cdot \lg \left( \log \left( \frac{1 - \sqrt{1 - T_P}}{1 + \sqrt{1 - T_P}} \cdot \frac{1 + \sqrt{1 - T_P / 1.05}}{1 - \sqrt{1 - T_P / 1.05}} \right) \right)$$



# Hubble diagram

**3-D space expansion speed:**  $\frac{dr}{dT} = \frac{dr}{dx} = \frac{d(x\theta)}{dx} = \theta \quad r = \theta \cdot x$

For a given angle  $\theta$ , variable  $x$ : constant  $\theta$

For a given radius  $x$ , variable  $\theta$ : proportional to  $\theta$  and to  $r \rightarrow$  **Hubble's law**

**Hubble diagram:** To discuss the recessive velocity of the **proper distance**

Tentatively provide that light speed has been constant.

Light propagated distance:  $LD_C = c \cdot (T_P - T_E) = c \cdot T_P \cdot (1 - T_{ER}) = c \cdot T_P \frac{z}{z+1}$

Multiply by  $(1+z)$  (**time dilation**):  $(1+z) \cdot LD_C = c \cdot T_P \cdot z \equiv k \cdot z \rightarrow$  proportional to  $z$

$\Delta t' = (1+z)\Delta t$  Light-curve width of supernovae: dilated by  $1+z$

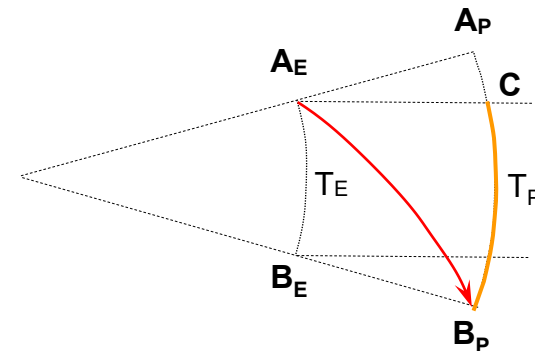
Conversion of LD to PD:

**A<sub>P</sub>-B<sub>P</sub>:** proper distance at  $T_P$  (“**present distance, PD**”)

**C-B<sub>P</sub>:** light propagated distance (LD)

$$n = T_P / T_E = 1 / T_{ER}$$

$$\frac{CB_P}{A_E B_E} = 1 + \frac{1}{2}(n-1) = \frac{1}{2}(n+1), \quad \frac{PD}{LD} = \frac{2n}{n+1} = \frac{2(z+1)}{z+2} = \frac{2}{1+T_{ER}}$$



## Adjusted magnitude of PD for Hubble diagram

In order to compare the **present distance** with reported Hubble diagrams, take the following value:

$$(1+z) \cdot PD = (1+z) \cdot \frac{2(z+1)}{z+2} \cdot LD \quad \text{or} \quad \frac{1}{T_{ER}} \cdot PD = \frac{1}{T_{ER}} \cdot \frac{2}{1+T_{ER}} \cdot LD$$

$\swarrow$ 
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 $\downarrow$

*Time dilation*
*LD-PD conversion*

In actual cosmological observation, in addition to the time dilation, K-correction is made to flux.

**K-corrections:** convert a measurement at redshift  $z$  to that in the rest frame at  $z = 0$ . Cross-filter adjustment in abs-Mag + difference in distance modulus between the two frames.

Theoretical part of K-corrections correspond to the LD-PD conversion.

**Adjusted magnitude of PD to  $z = 0.05$ ,**  $DM_{0.05}^{adj}$ : Add **Time dilation** and **LD-PD conversion** to  $DM_{0.05}$

$$DM_{0.05}^{adj}(T_{ER}) = 5 \cdot (\lg LD(T_{ER}) + \lg(1/T_{ER}) + \lg(2/(1+T_{ER}))) - \lg LD(1/1.05) - \lg 1.05 - \lg(2 * 1.05/2.05)$$

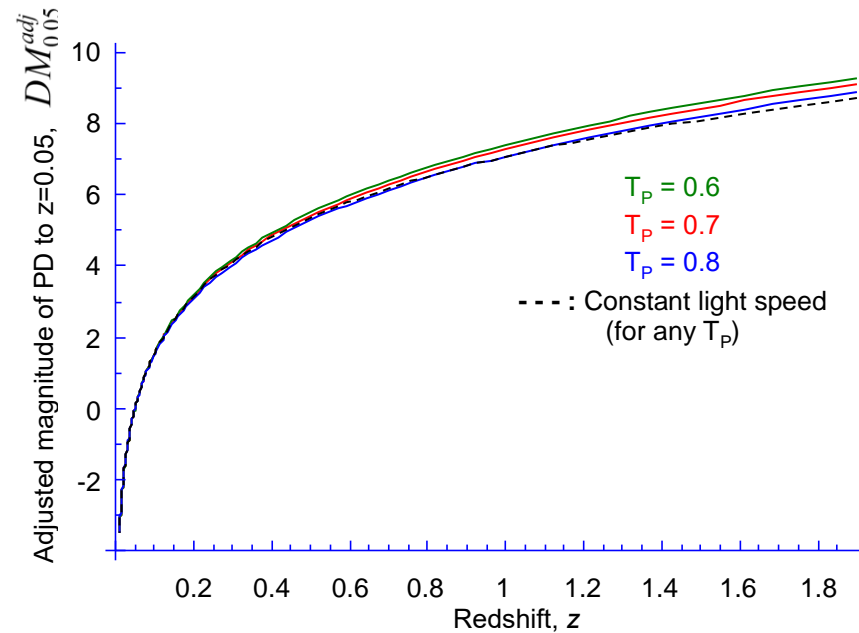
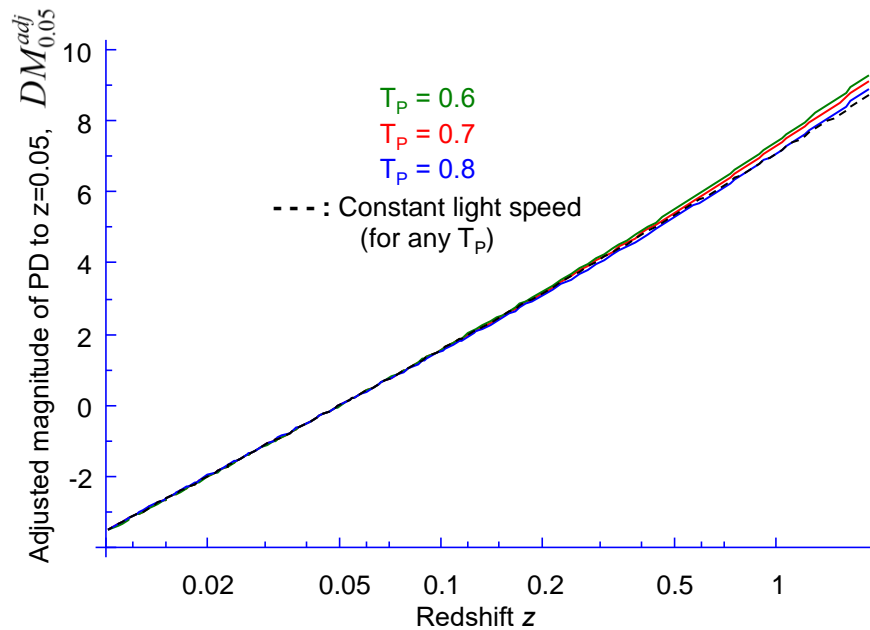
$$= 5 \cdot (\lg LD(T_{ER}) - \lg(T_{ER}) - \lg(1+T_{ER}) - \lg LD(1/1.05) - 2 \cdot \lg 1.05 + \lg 2.05)$$

$$DM_{0.05}^{adj}(T_{ER}) = 5 \cdot \lg \left( \log \left( \frac{1 - \sqrt{1 - T_P}}{1 + \sqrt{1 - T_P}} \cdot \frac{1 + \sqrt{1 - T_P T_{ER}}}{1 - \sqrt{1 - T_P T_{ER}}} \right) \right) - 5 \cdot \lg \left( \log \left( \frac{1 - \sqrt{1 - T_P}}{1 + \sqrt{1 - T_P}} \cdot \frac{1 + \sqrt{1 - T_P / 1.05}}{1 - \sqrt{1 - T_P / 1.05}} \right) \right)$$

$$- 5 \cdot \lg T_{ER} - 5 \cdot \lg(1 + T_{ER}) - 10 \cdot \lg 1.05 + 5 \cdot \lg 2.05$$

# Hubble diagram of adjusted PD vs redshift

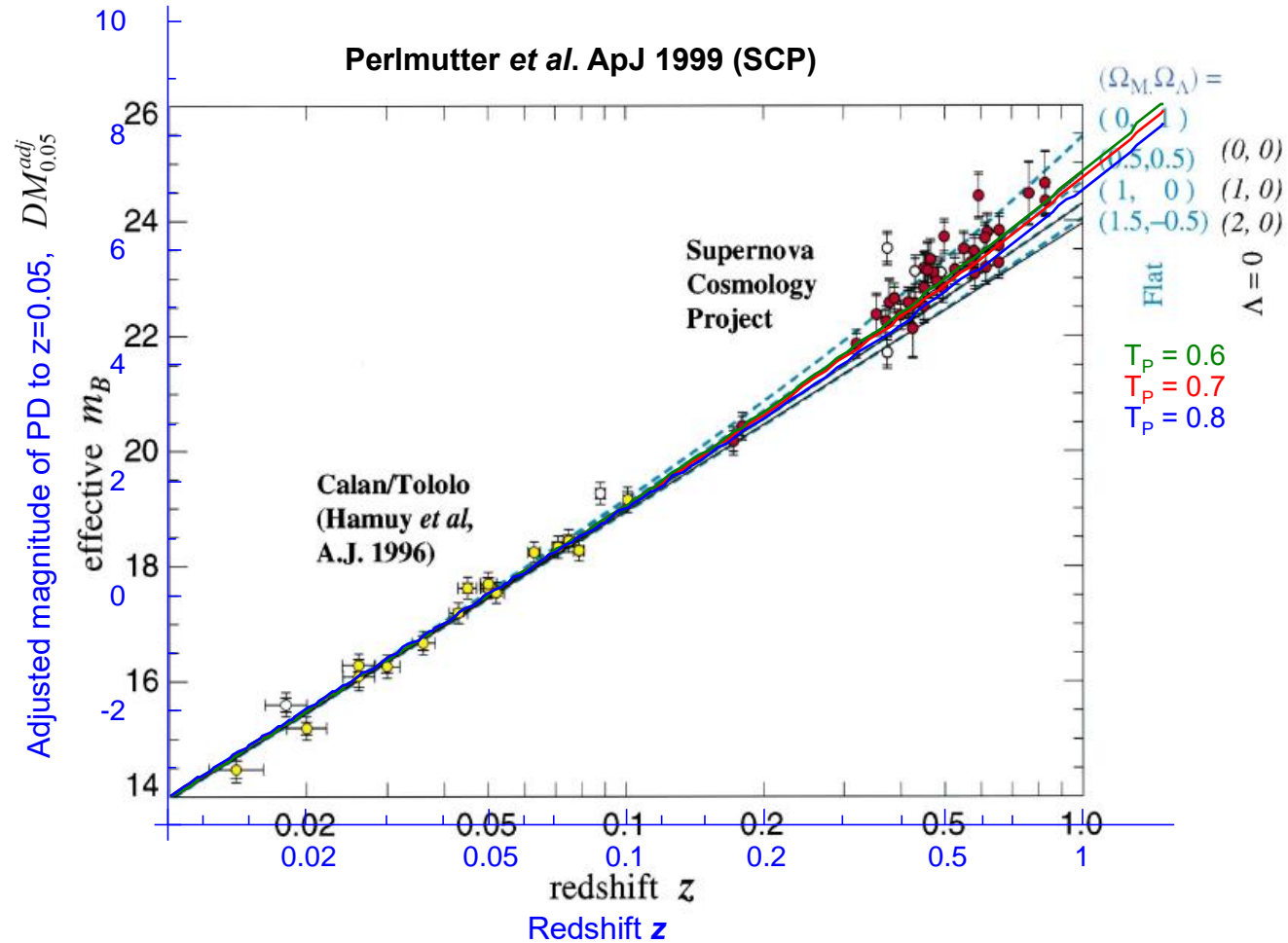
The adjusted magnitude of PD to  $z = 0.05$ ,  $DM_{0.05}^{adj}$  versus the redshift  $z = 1/T_{ER} - 1$



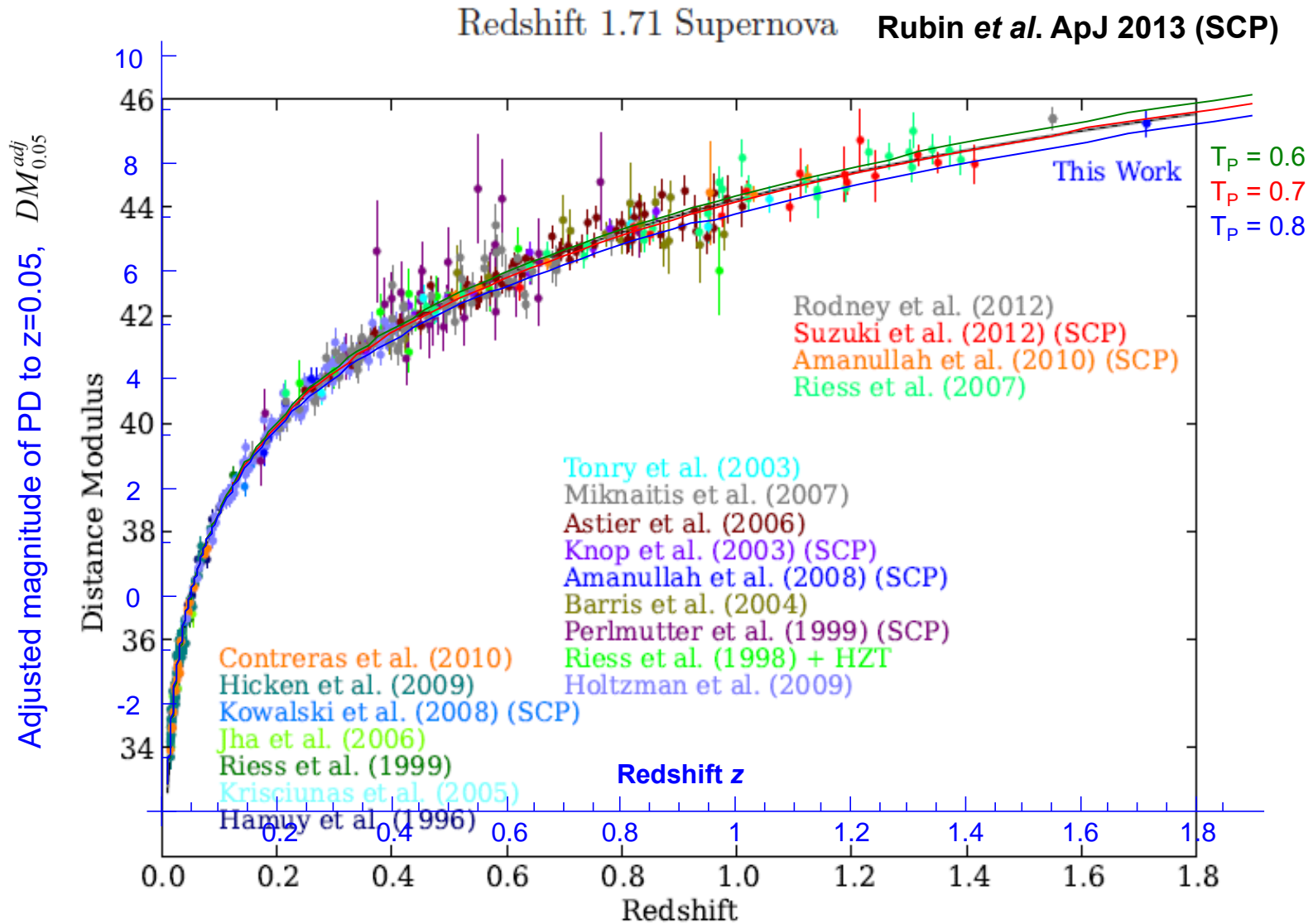
-----: reference  $DM_{0.05}^{adj}$  based on  $LD_C = c \cdot T_P(1 - T_{ER})$  subject to constant light speed

## 12. Comparison with Hubble diagrams from the SCP

Superimposition on the Hubble diagram by Perlmutter et al,  $z$  in a logarithmic scale



Superimposition on the latest Hubble diagram from the SCP,  $z$  in a uniform scale



## Conclusion of comparison with the SCP

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- Observed redshift is from values of the present atom.
- Light propagated distance LD is equal to the luminosity distance.
- Ratio of converting LD to the present distance PD (proper distance at present) is given.
- The frame-conversion part of the K-correction corresponds to the LD-PD conversion.
- Magnitude of  $(1 + z) \cdot PD$  to  $z = 0.05$  was compared with reported Hubble diagrams from the SCP, which were time-dilated by  $1 + z$  and K-corrected.
- Superimposition on the reported Hubble diagrams from the SCP showed an excellent fit. The graph for the Present Time  $T_p = 0.7$  very closely overlaps the line of flat  $\Omega_m = 0.27$   $\Lambda$ CDM universe that Rubin et al concluded as the best fit.

The **expansion of universe** is **not** accelerating.

Decelerating by the Original time

Constant by the Observed Time.